Introducing Differential Equations to Social Scientists

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Computationally Intensive Modeling of Social Interaction
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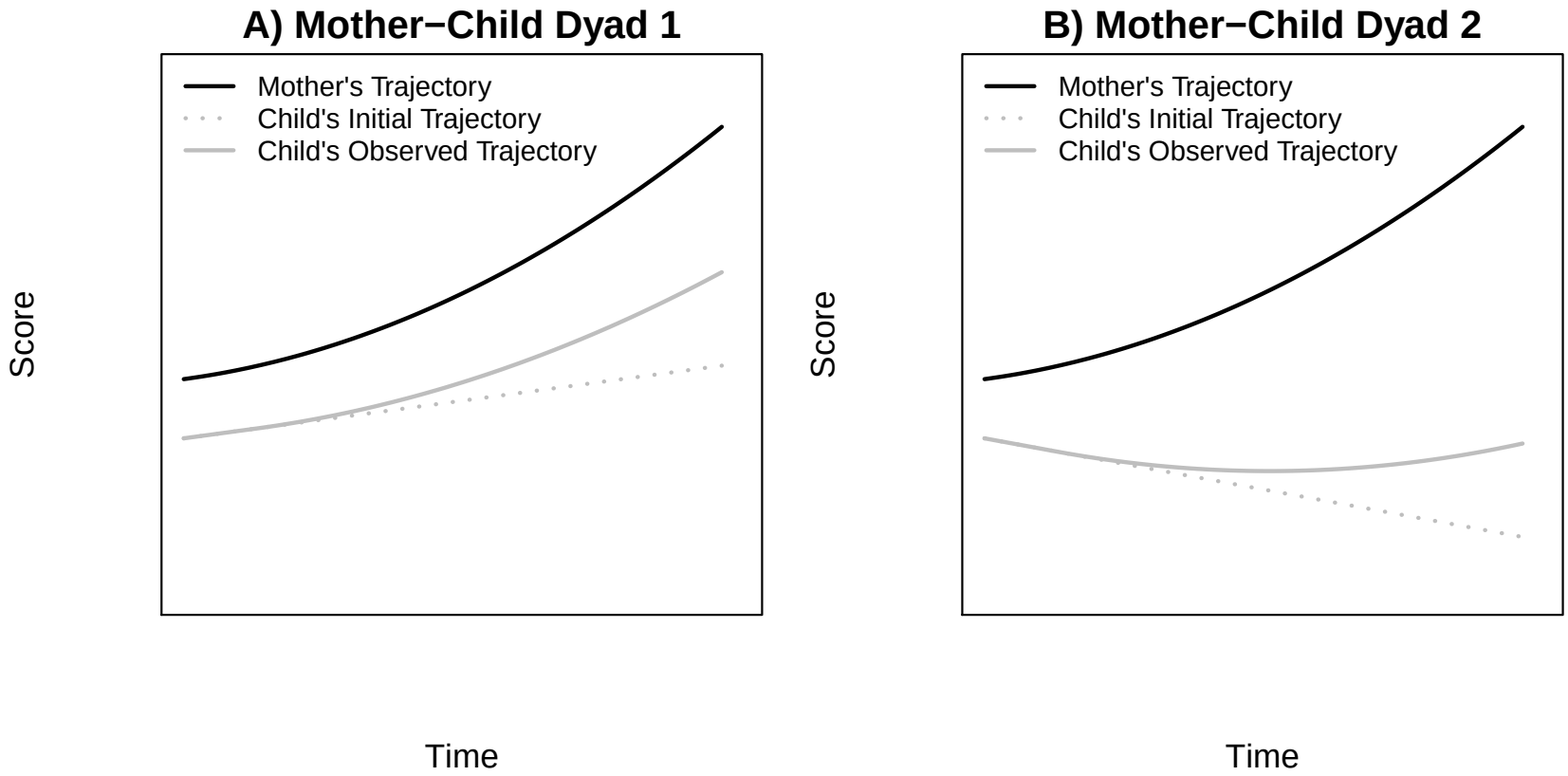
Introduction

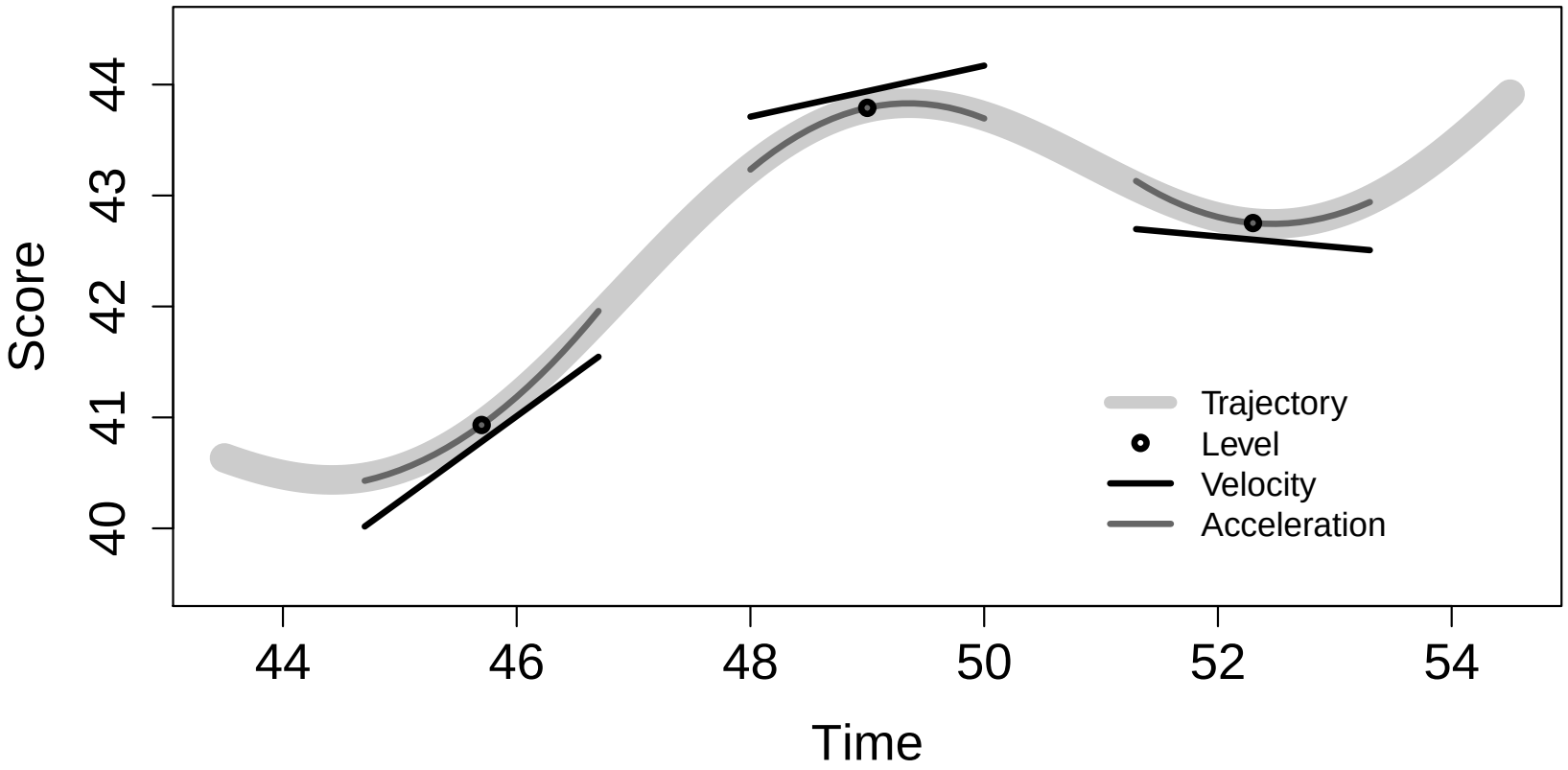
- Repeated intraindividual observations, typically on continuous variables, significant proportions of measurement & process/dynamic error, missing data, unequal lags
- Differential equations (characterize intraindividual variability, interesting parameters, parameters that are independent of sampling rate, variation in lags between observations...)
- The Challenge

- Three ideas:
  - "Related Change"
  - Translating theory into testable models
  - Math 115 and Math 866

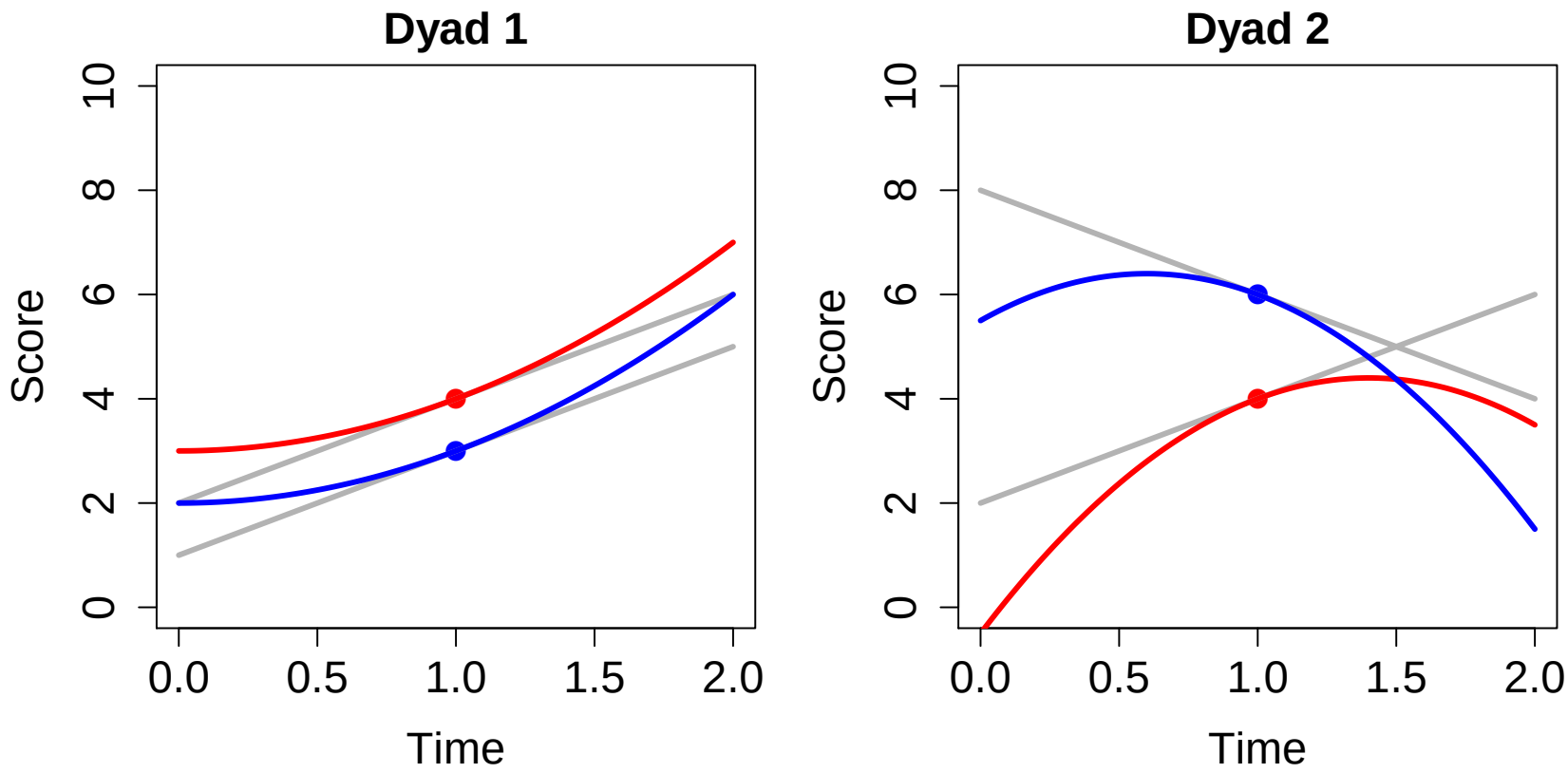
Related Change



Describing Change



Non–tandem Changes



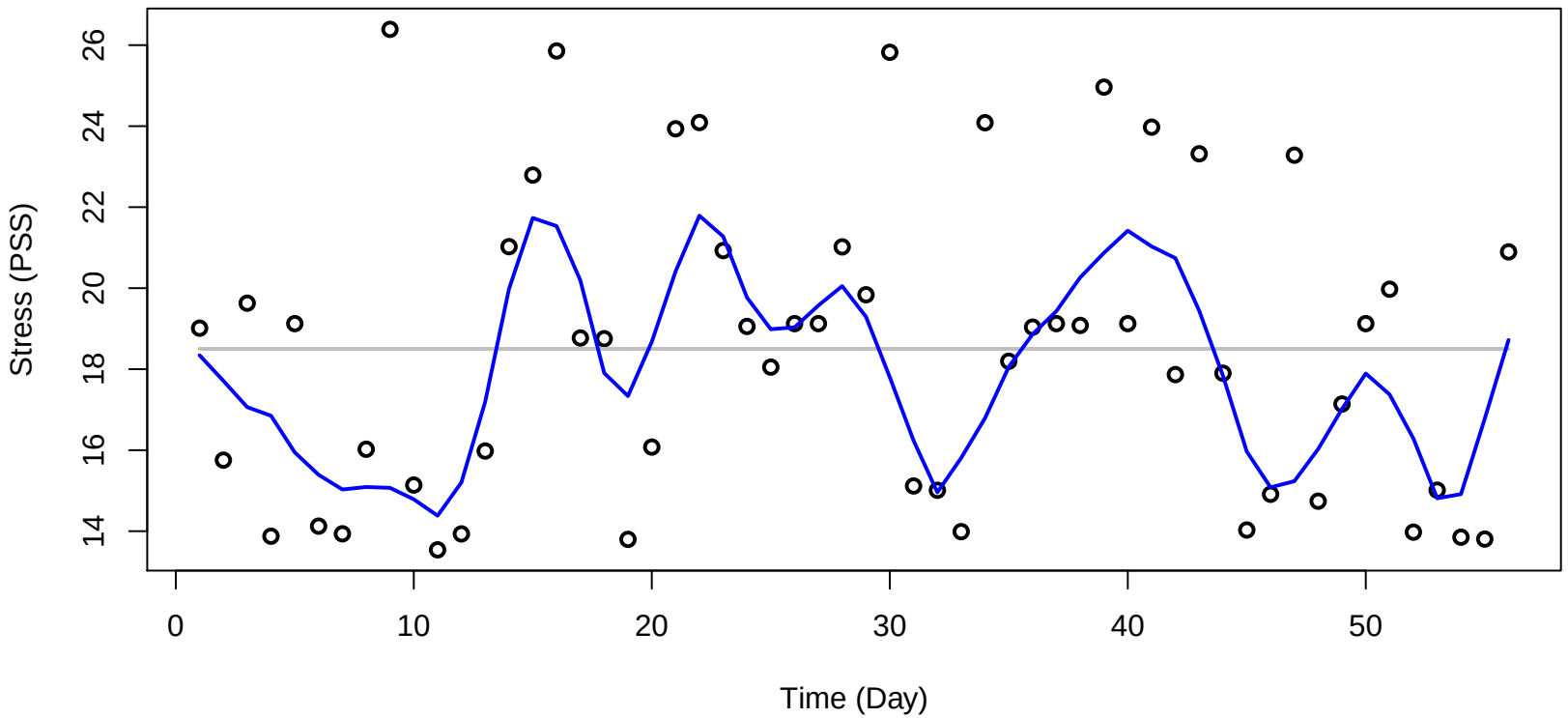
Level, Velocity, Acceleration

Related Change

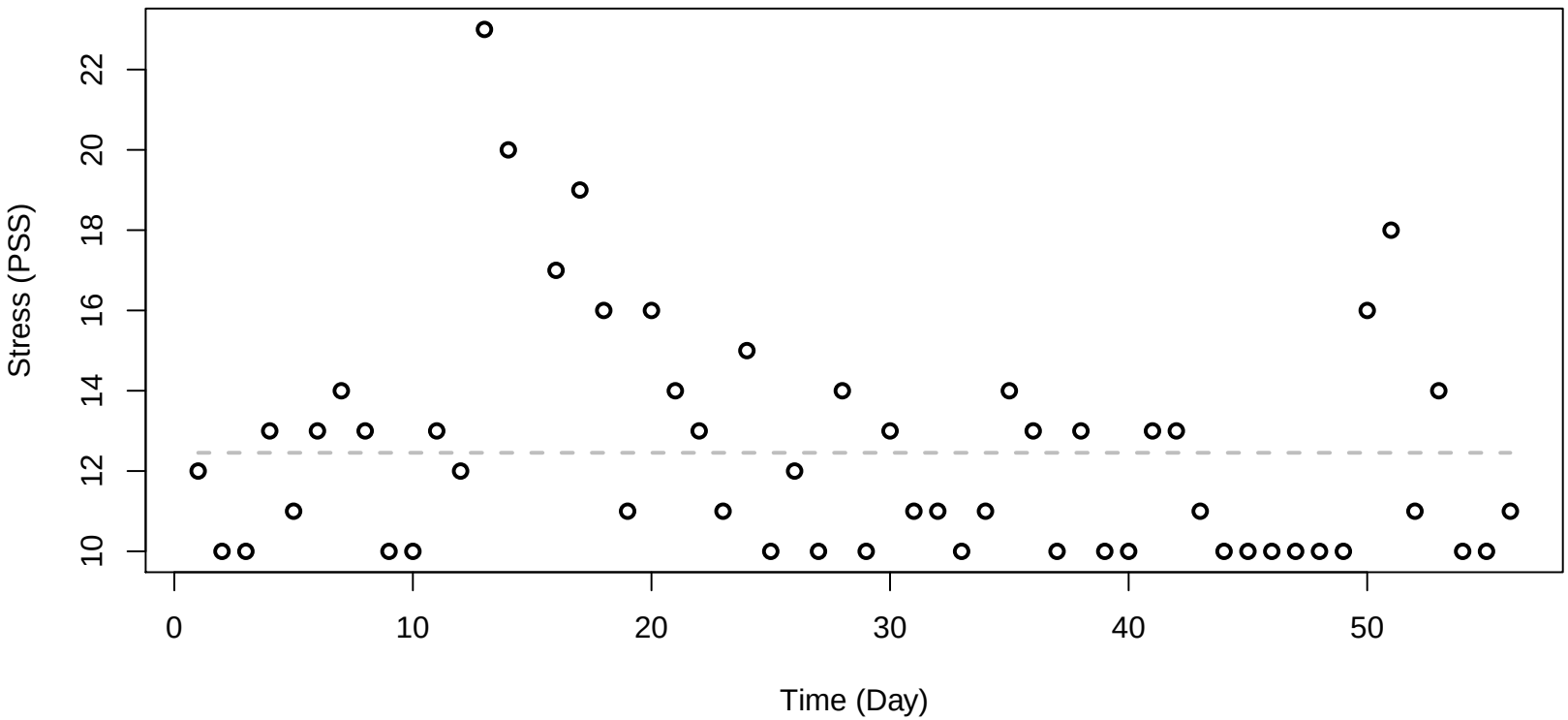
		Construct 2		
		Level	**Velocity**	**Acceleration**
Construct 1	**Level**	Level-Level: Are high *levels* of maternal depression observed with high *levels* of child behavior problem?		
	Velocity	Velocity-Level: Is a mother's *level* of depression, regardless of whether her symptoms are changing or not, related to the rate at which her child's behavior problems increased or decreased (*velocity*)?	Velocity-Velocity: Does the rate at which mother's depressive symptoms increase or decrease (*velocity)* predict the rate at which her child's behavior problems increase or decrease (*velocity*)?	
	Acceleration	Acceleration-Level: Does the mother's *level* of depression predict increases or decreases in the rate of change of her child's behavior problems (changes in velocity, *acceleration*)?	Acceleration-Velocity: Does the rate at which a mother's depression improves or worsens (*velocity)* predict increases or decreases in the rate of change of her child's behavior problems (changes in velocity, *acceleration*)?	Acceleration-Acceleration: Do increasing or decreasing rates of change in maternal depression (changes in velocity, acceleration) predict increases or decreases in the rate of change of her child's behavior problems (changes in velocity, *acceleration*)?

From an unpublished paper by P. R. Deboeck, J. S. Nicholson, C. D. Kouros, J. Garber & T. D. Little

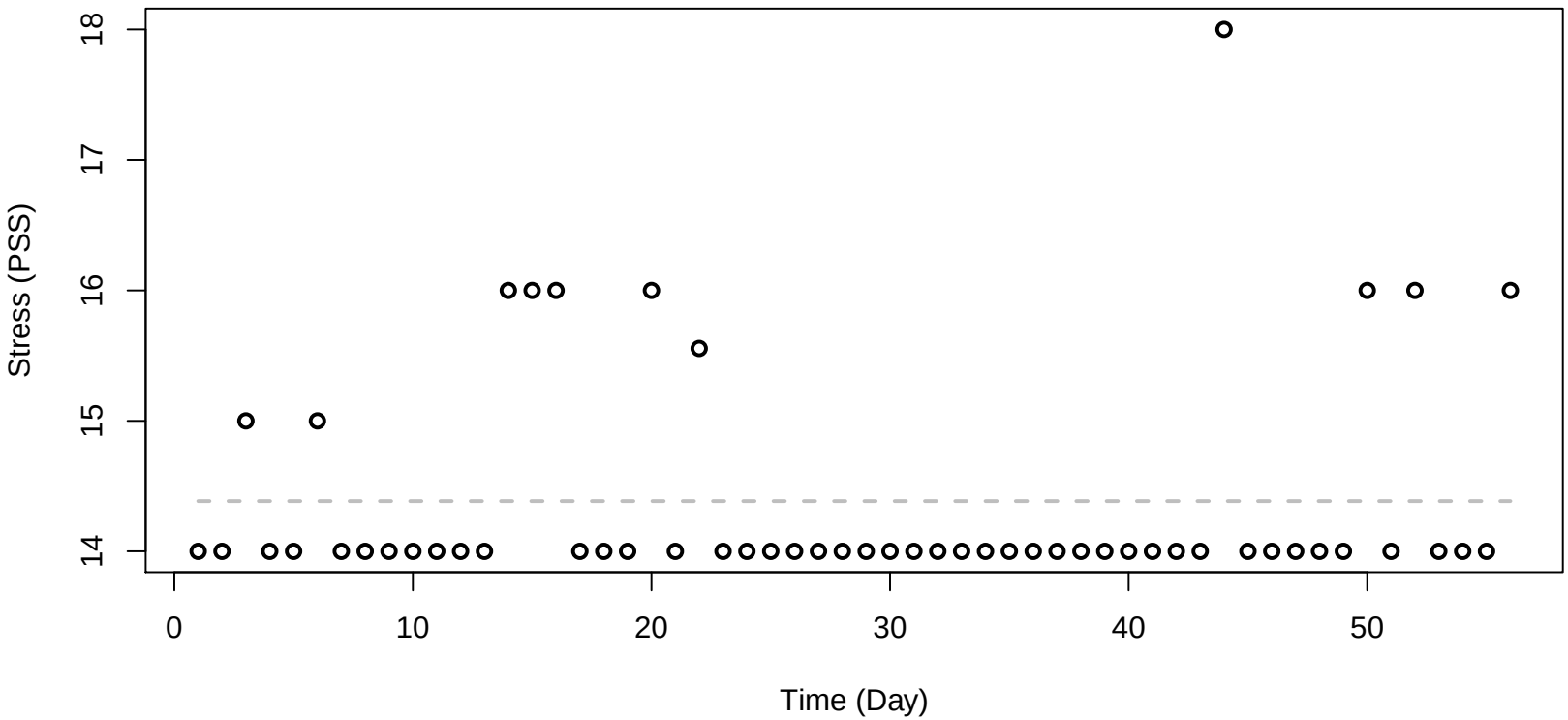
Stress Data



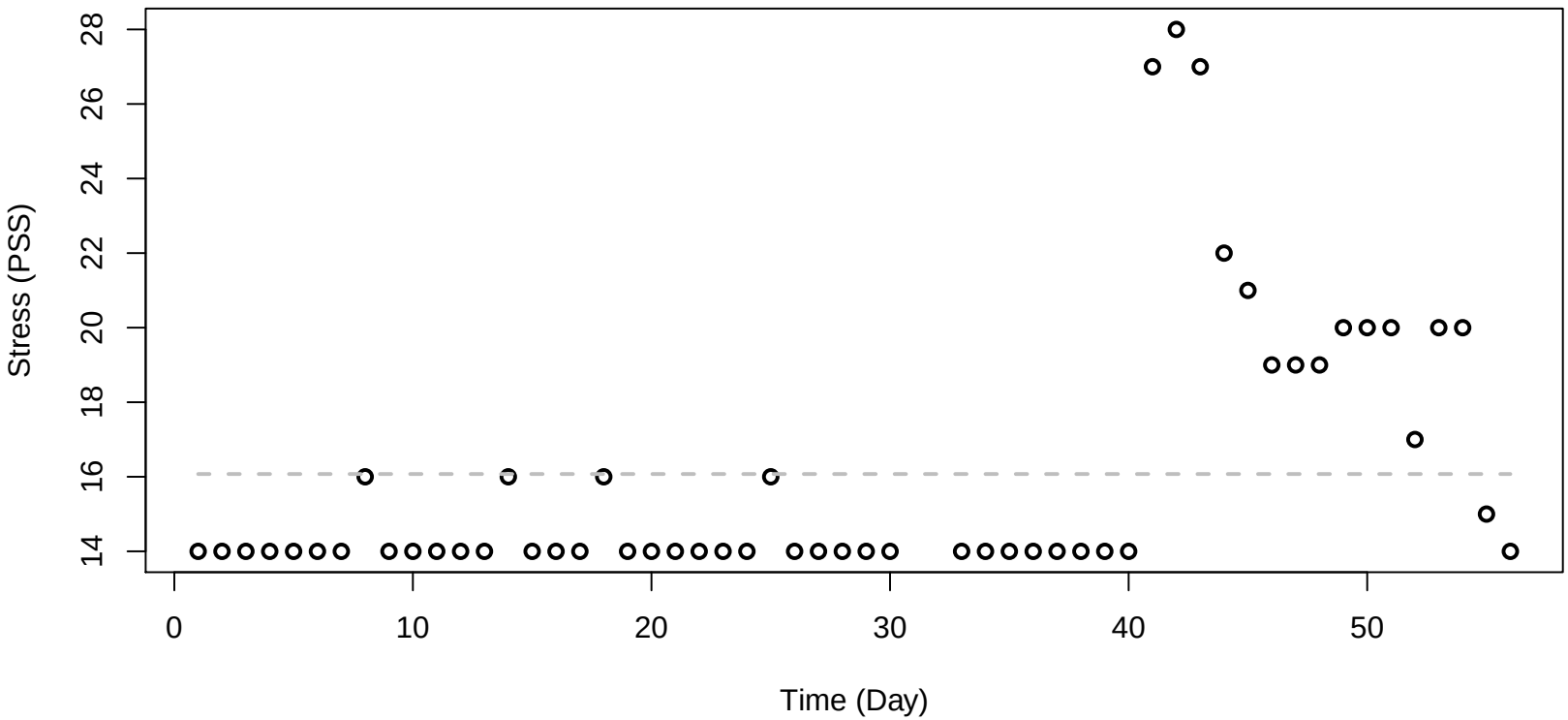
Stress Data 1



Stress Data 2



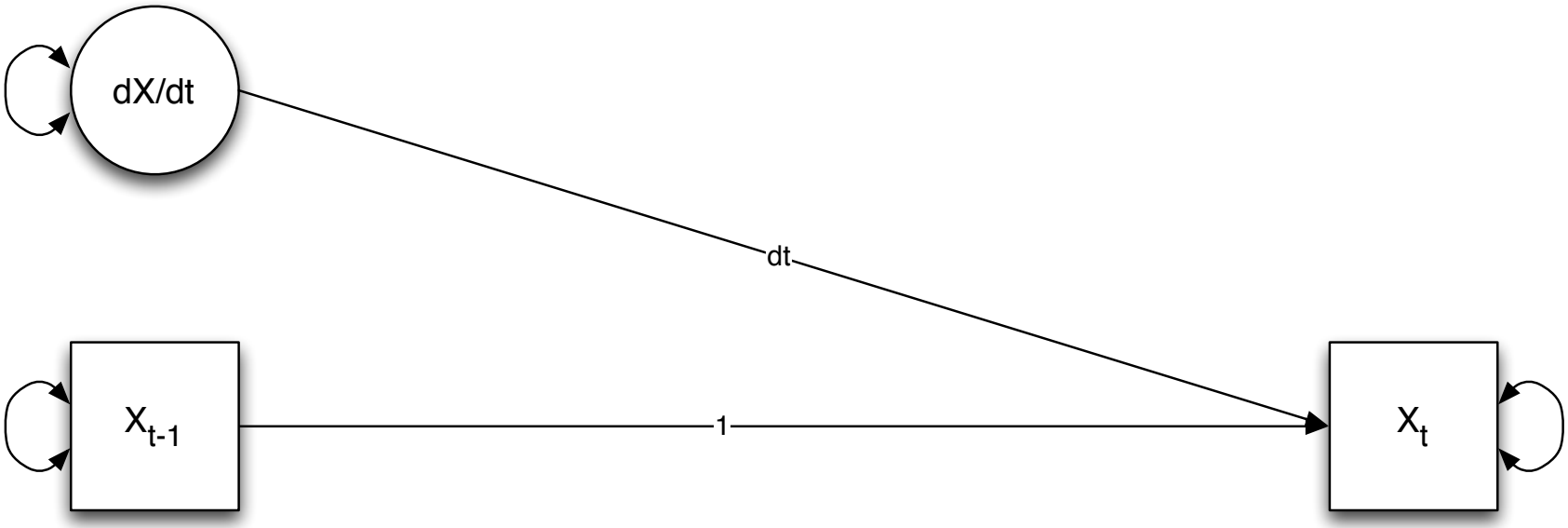
Stress Data 3



Physical Analogy

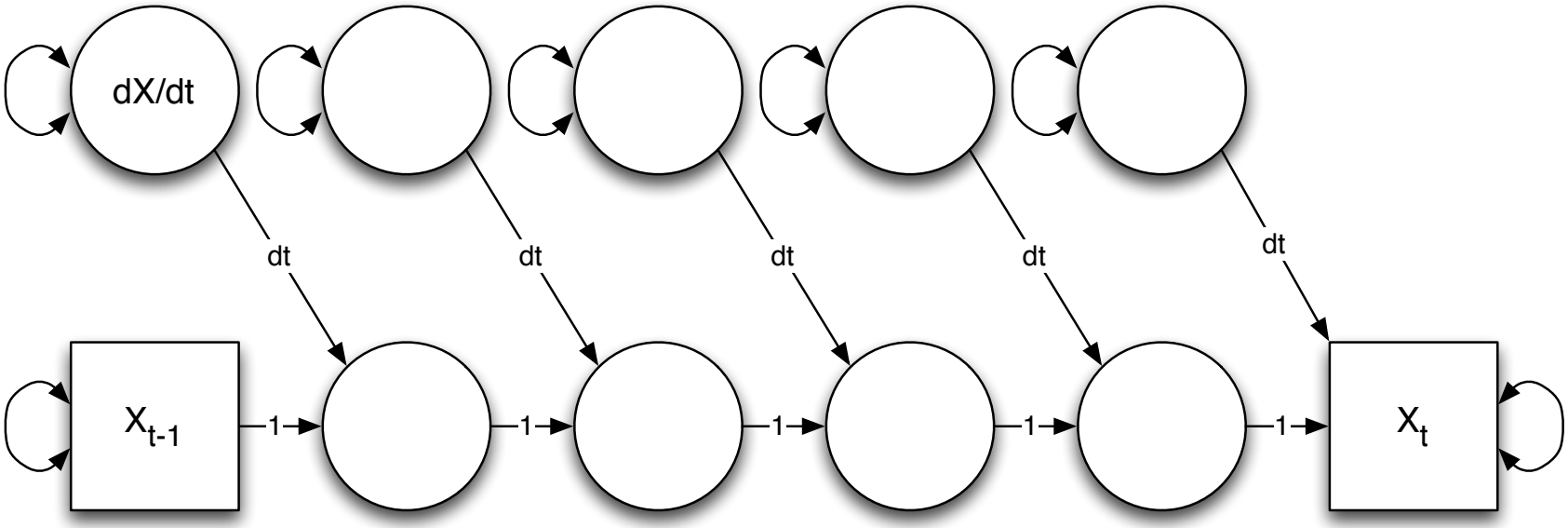
Physical Analogy

Estimating Derivatives

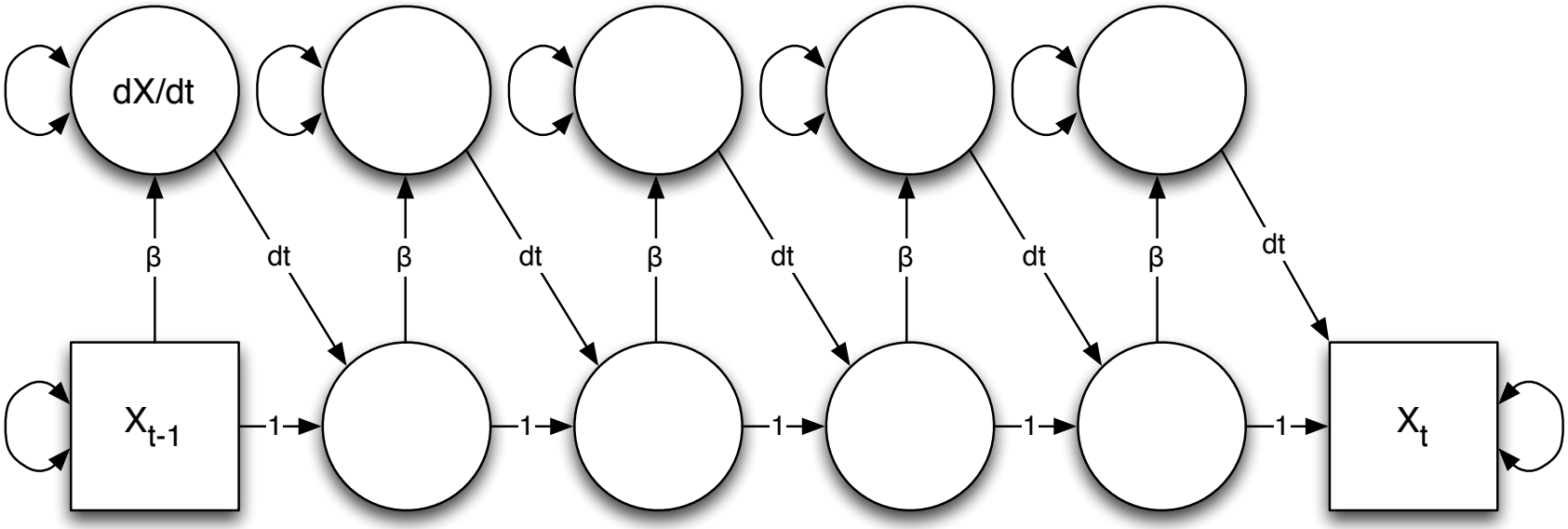


- $x_t = x_{t-1} + \left(\frac{dx}{dt}\right) dt$
- Everyone has one estimate of velocity

Continuous Time Models

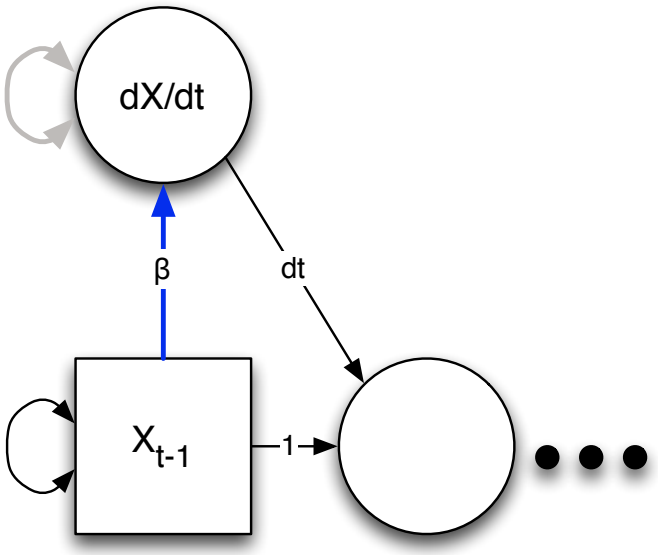


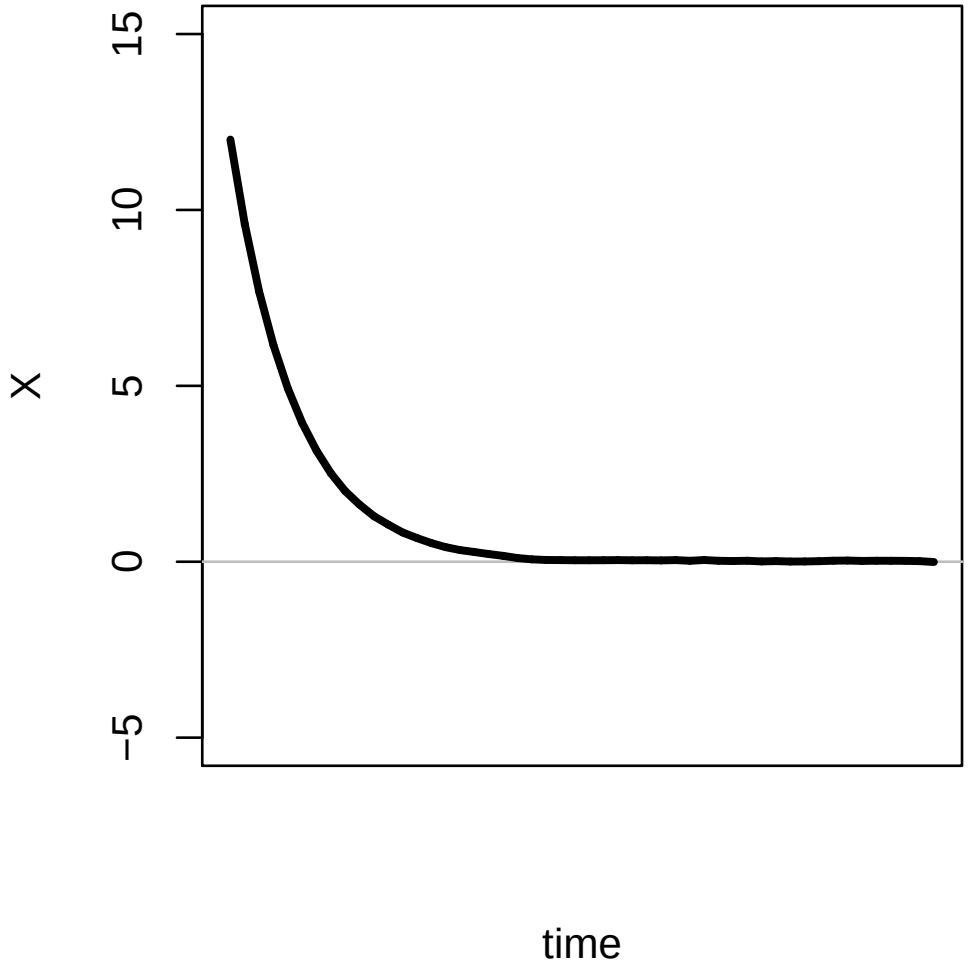
Another Model



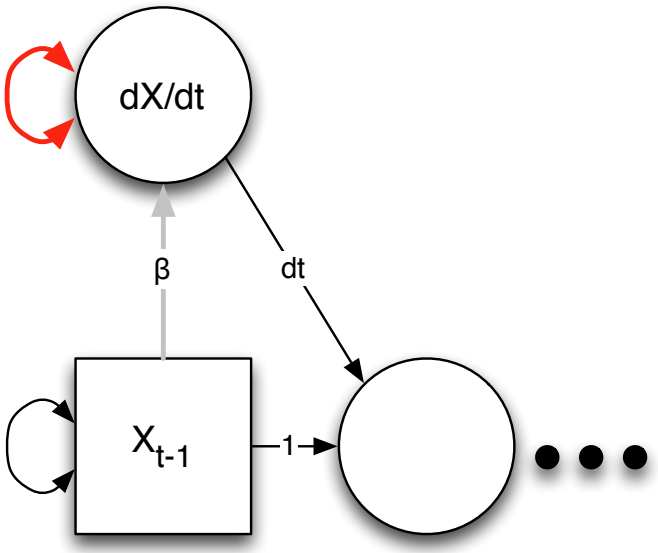
- $\frac{dX}{dt} = \beta X + \varepsilon$
- First–order stochastic differential equation model

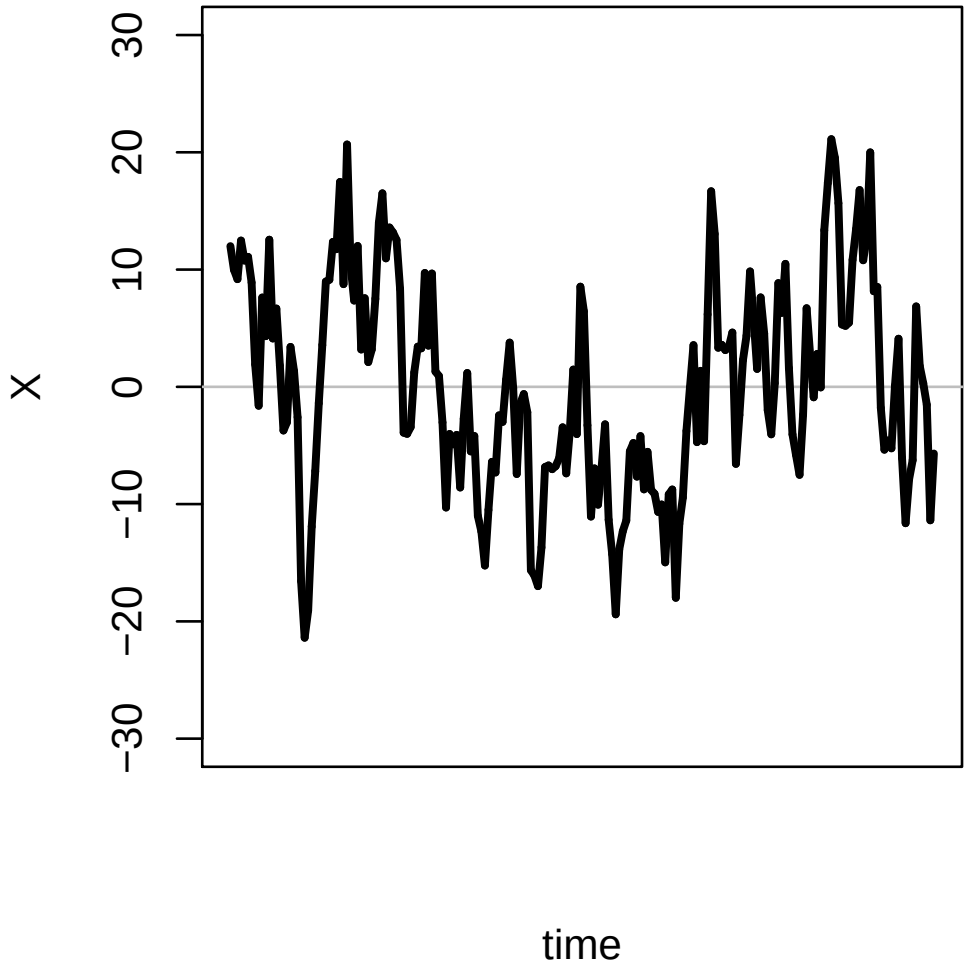
$$\frac{dX}{dt} = \beta X + \varepsilon$$





$$\frac{dX}{dt} = AX + \varepsilon$$





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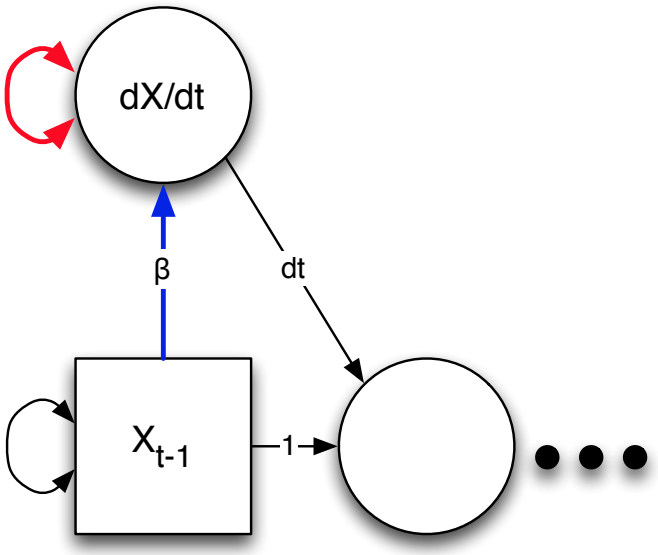

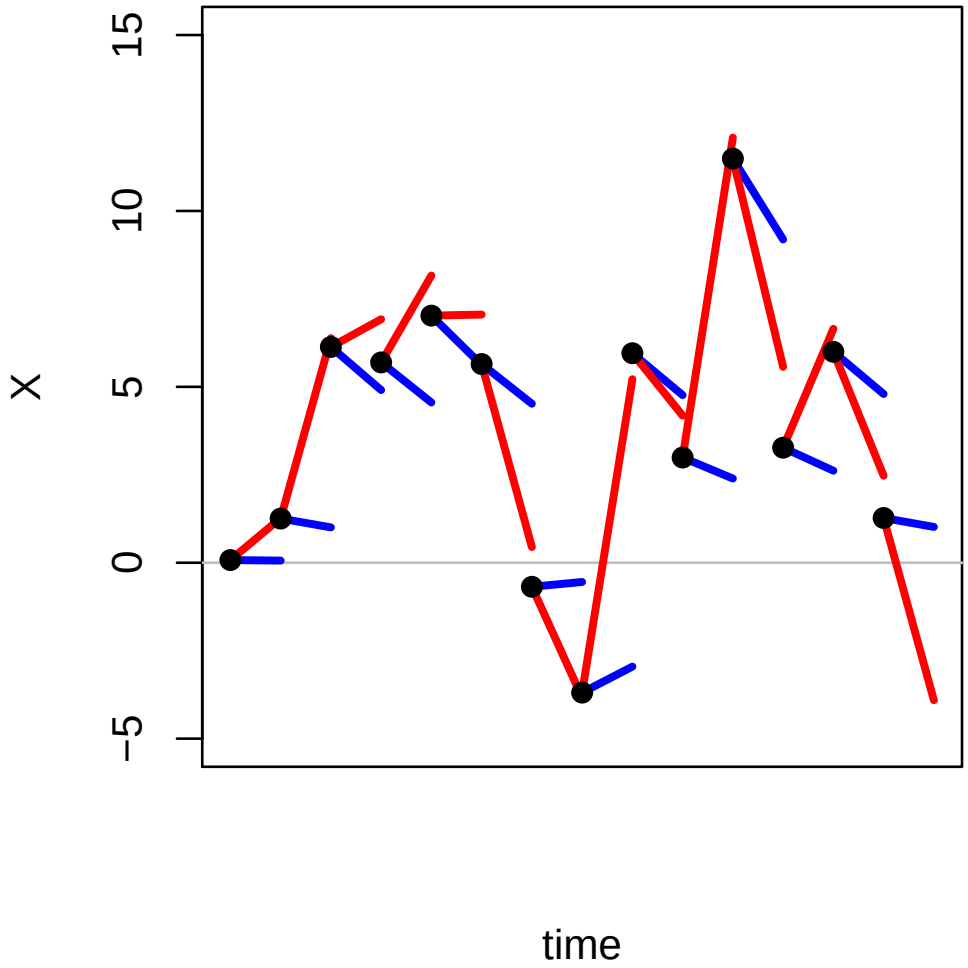

Summary

- Three ideas:
  - "Related Change"
  - Translating theory into testable models
  - Math 115 and Math 866
- Deception Debrief:
  - Derivatives
  - Differential Equations
  - Stochastic Differential Equations