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Displays that facilitate performance of multifrequency ratios during motor-respiratory coordination

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ABSTRACT

A large number of ratios between movement and breathing are possible, but only a small number have been performed during exercise. The purpose of this study was twofold: (1) to investigate displays that might facilitate the performance of other ratios; and (2) to test predictions from the sine circle map and continued fractions in a model motor-respiratory task in which participants coordinated arm movement and breathing. Displays consisted of either real-time feedback or a template (non-feedback). The accuracy of ratio performance was significantly greater with the template in which the number and relative positioning of movements and breaths was depicted, compared to with real-time feedback. Across displays, the stability of ratio performance conformed to principles of the sine circle map and was significantly greater for ratios with longer continued fractions. Therefore, the motor-respiratory repertoire can be expanded by increasing participants' understanding of the pattern to be performed, but performance is constrained by general dynamical principles.

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1. Introduction

Motor-respiratory coordination (MRC), the synchronization of movement and breathing, has been observed in a wide range of exercises including walking (van Alphen & Duffin, 1994), running (Bernasconi & Kohl, 1993; Bramble & Carrier, 1983), cycling (Garlando, Kohl, Koller, & Pietsch, 1985), rowing (Mahler, Hunter, Lentine, & Ward, 1991; Mahler, Shuhart, Brew, & Stukel, 1991) and wheelchair propulsion (Amazeen, Amazeen, & Beek, 2001). Coordination is captured by the frequency ratio—the number of movement cycles produced per breath. Theoretically, a large number of frequency ratios can be produced, given the upper and lower boundaries on normal movement and breathing frequencies. However, only a small number have been observed: 1:2 (during rowing), 1:1, 2:1, 3:1, 4:1, 5:1, 6:1, 8:1, 10:1, 3:2 and 5:2. Larger-integer, complex ratios (e.g., 5:4) have not been observed and are predicted to be less stable in a model called the sine circle map (Peper, Beek, & van Wieringen, 1995a, 1995b; Treffner & Turvey, 1993; Villard, Casties, & Mottet, 2005). In the present study, displays were designed to facilitate performance of any (simple or complex) motor-respiratory frequency ratio through enhancement of both perception and comprehension features of displays. The performance of ratios not typically observed will allow us to test the application of the sine circle map to MRC.

1.1. The Lissajous display

To date, displays have not been used to facilitate MRC. However, Lissajous displays have been used to facilitate bimanual coordination by reducing two movement trajectories into the production of a single collective shape on a computer screen. This shape varies with different ratios and phase offsets (the relative difference in movement cycle locations). For example, the Lissajous display for 2:1—characterized by alternating epochs where muscles flex and extend together and then flex and extend in alternation—is a simple V shape (see Fig. 1A) (Swinnen, Dounskaia, Walter, & Serrien, 1997). Use of Lissajous displays has increased the accuracy and stability of 1:1 (Hurley & Lee, 2006; Lee, Swinnen, & Verschueren, 1995; Wenderoth & Bock, 2001) and 2:1 (Swinnen et al., 1997) coordination compared to performance with only proprioceptive information. However, these results will not necessarily translate to other ratios. A comparison between idealized Lissajous displays (Fig. 1) for 2:1 (Panel A) and 5:3 (Panel B) illustrates the problem. The 5:3 Lissajous display is a complex web of lines. This increased

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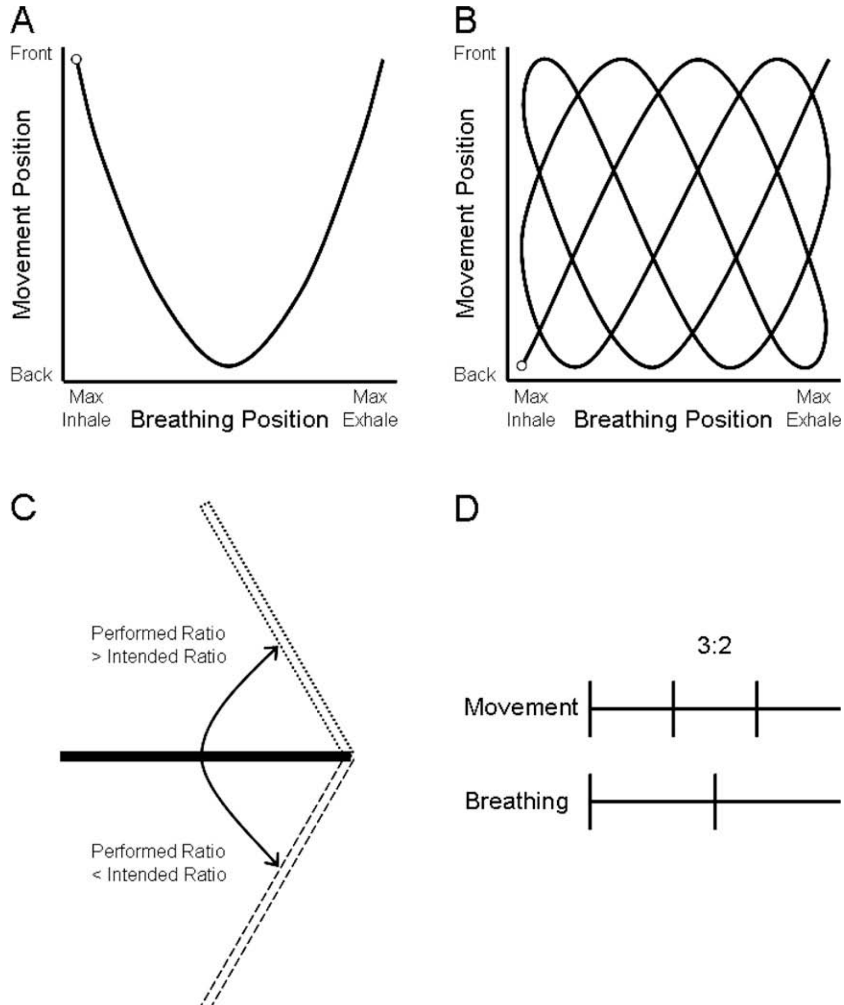

Fig. 1. Idealized Lissajous displays appear simple for 2:1 (A) and more complex for 5:3 (B). In the perceptually manipulated display a line rotates between a maximum of 60° (dotted line) and a minimum of −60° (dashed line) where horizontal (black) represents accurate ratio performance (C). In the static performance template there are two horizontal lines, one for movement and one for breathing, with vertical hash marks specifying either forward arm movements or inhalations. A 3:2 ratio is depicted (D).

complexity is likely to make Lissajous displays for more difficult multifrequency ratios challenging to use.

1.2. Displays for multifrequency motor-respiratory coordination

Increased complexity for difficult multifrequency ratios motivates the need to produce more accessible displays. One approach is to increase the compatibility between MRC and perceptual changes in a display. Perception-performance compatibility has been examined previously (Byblow, Chua, Bysouth-Young, & Summers, 1999; Wilson, Collins, & Bingham, 2005). In one study, the participants controlled a ball to produce certain relative phases with a computer-controlled ball (Wilson et al., 2005). The participant-controlled ball moved left to right on a monitor in response to either left to right (compatible with ball movement) or circular (incompatible with ball movement) joystick movement. In a bimanual coordination study, a row of five light-emitting diodes was illuminated in the same (compatible) or opposite (incompatible) direction as dominant hand movement (Byblow et al., 1999). In both cases, relative phase variability was lower with the compatible manipulation.

In the present study, the task was to coordinate breathing and sagittal arm movement. A compatibility issue for sagittal arm movement is that people tend to interpret the forward–backward direction in egocentric space as the upward–downward direction on a vertical map (Shepard & Hurwitz, 1984). This orientation, which can be labeled a *natural mapping*, suggests that people take advantage of physical analogies to facilitate their understanding of objects (Norman, 1988). For the purposes of the present study, it suggests that upward–downward motion of a ball in a display is compatible with sagittal arm movement. In contrast, breathing, in an egocentric sense, is analogous to inflating and deflating a balloon with each inhalation and exhalation. This observation suggests that a more compatible mapping for breathing is expansion–contraction of a balloon rather than upward–downward motion of a ball. In the present study, we tested different computerized displays to probe this compatibility issue: (1) a ball–ball display (compatible for movement, less compatible for breathing); (2) a balloon–balloon display (less compatible for movement, compatible for breathing); and (3) a ball–balloon display (compatible for both movement and breathing).

A characteristic of the ball and/or balloon displays is that feedback is veridical. Animated perceptual motion corresponds directly to a participant's movement or breathing. Feedback of this type may be too difficult because accurate ratio performance requires that the participants count the number of cycles for each display component. Another approach is to manipulate the perceptual feedback provided. Previous studies suggest that perceptually manipulated feedback can improve performance. In bimanual coordination, the participants were instructed to maintain inphase between two flags or dots by moving their hands in a 4:3 ratio (Mechsner, Kerzel, Knoblich, & Prinz, 2001) or various phase relationships (Amazeen, Da Silva, & Amazeen, 2008; Tomatsu & Ohtsuki, 2005). The participants accurately performed 4:3 (Mechsner et al., 2001) and improved performance on more difficult phase relationships relative to either veridical (Tomatsu & Ohtsuki, 2005) or antiphase (Amazeen et al., 2008) feedback. In the current study, we further simplified feedback by representing perfect coordination as a horizontal line, deviations from which specified the magnitude and direction of error (see Fig. 1C). When using that display, the participant's task—to keep the line horizontal—was simplified.

If an understanding of the ratio is necessary for successful performance, then manipulating feedback alone so as to make the display perceptually simpler may not produce better performance. One alternative is that participants use proprioceptive information to guide their actions to match the requirements of a static performance template. The task for participants is perceptually burdensome but conceptually simplified. That is, they must have adequate proprioceptive information to evaluate whether they are performing the ratio correctly, but the motor-respiratory relations that specify the ratio are identified for them in the template. A performance template for 3:2 is presented in Fig. 1D. The template is composed of horizontal lines for movement and breathing with hash marks that represent forward arm movements or inhalations. Any ratio can be represented by changing the number of hash marks on each line. Although performance templates provide no feedback, they have been beneficial to the learning of both 5:2 and 5:3 in MRC (Hessler & Amazeen, submitted for publication) and the training of multifrequency ratios in bimanual coordination (Summers, Rosenbaum, Burns, & Ford, 1993).

1.3. The sine circle map

In addition to examining the potential benefits that displays have for ratio performance, we will test the hypothesis that MRC is constrained by the natural dynamics specified in the sine circle map (Bak, 1986; González & Piro, 1985; Hardy & Wright, 1938). The sine circle map identifies regions of stability, or frequency locking, in the coupling of any two oscillators (Bak, 1986). It identifies, at intervals established by the cycle time of one oscillator (e.g., breathing), the phase angle θ_n of the second oscillator (e.g., arm movement), at time n:

$$\theta_{n+1} = \theta_n + \Omega + \frac{K}{2\pi}\sin(2\pi\theta_n)(\mathrm{mod}1). \tag{1}$$

The phase angle of the arm at the next time step, θ_{n+1}, is determined by three factors: (1) its phase angle at the previous time step; (2) the bare winding number Ω, which is the ratio of uncoupled frequencies (often the required ratio in an experiment); and (3) a nonlinear coupling function of strength K, which theoretically varies with various (physiological, perceptual, and conceptual) constraints. Predictions about the observed ratio, or winding number, W, are made by estimating the average shift in θ per iteration as $n \to \infty$. In the absence of coupling ($K = 0$), $W = \Omega$, and can thus be rational or irrational. However, in the presence of some coupling ($0 < K < 1$), W is rational and varies as a function of Ω and K.

Two graphical depictions (Arnold tongues, Farey tree) facilitate interpreting the influence of Ω and K on W. Regions of stability are easy to identify in the white Arnold tongues (Arnold, 1965) shown in Fig. 2. Regions of instability are the black spaces between the tongues. All ratios are on the interval from 0 to 1, so that, for example, 2:1 = 1:2 = 0.5. In the present paper, we will generally adhere to the conventional format in the motor-respiratory literature

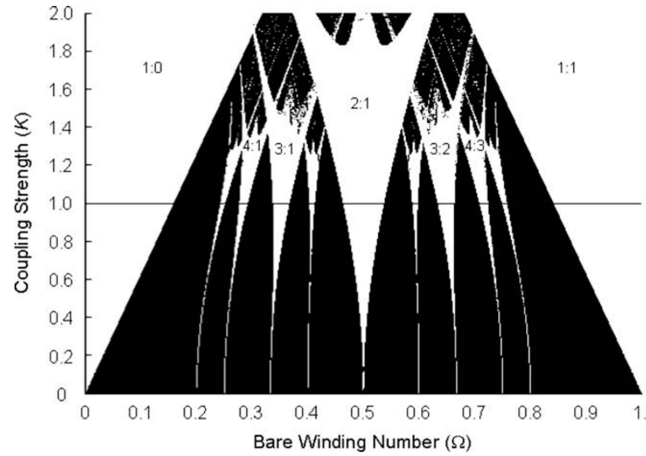

Fig. 2. The Arnold tongue regime diagram simulated via iteration of the sine circle map. The bare winding number, Ω, is the ratio between the forced and unforced oscillators, and coupling strength, K, represents the degree of pushing and pulling between the oscillators. Irrational ratios that fall within Arnold tongues (white) resonate toward the specific rational ratio corresponding to each tongue. Only tongues for ratios in which both integers are less than six are depicted. Larger resonances are labeled with the corresponding frequency ratio. Above $K = 1$, all Arnold tongues overlap, and behavior chaotically switches among rational ratios.

(e.g., Amazeen et al., 2001; Bramble & Carrier, 1983) of identifying movement-to-breathing ratios (usually $p > q$). Some tongues are wider than others. Wider tongues correspond to greater stability (e.g., 2:1). Empirically, this is demonstrated in two circumstances: when the same ratio is performed (1) despite the intention to produce different ratios (variations in Ω) or (2) across variations in the overall frequency of ratio performance (variations in K).

The Farey tree demonstrates, via a simple mathematical rule, the size ordering of the Arnold tongues. The first five levels of the Farey tree are depicted in Fig. 3. The lowest level (Level 0) contains the *parent ratios*, 1:0 and 1:1, the numerical boundaries of Ω. Higher levels are produced through Farey summation, $(p + p')/(q + q')$, of adjacent ratios at lower levels, p:q and p':q'. For example, Farey summation of 1:0 and 1:1 on Level 0 produces the *child ratio* 2:1 at Level 1. The same process can be used to generate an entire inverted "tree" of ratios. When applied to bimanual coordination (de Guzman & Kelso, 1991; Deutsch, 1983; Haken, Peper, Beek, & Daffertshofer, 1996; Peper, Beek, & van Wieringen, 1991; Peper et al., 1995a, 1995b; Treffner & Turvey, 1993) and MRC (Amazeen et al., 2001; Hessler & Amazeen, submitted for publication; Villard et al., 2005), the prediction and finding is that lower-level ratio performance is more stable than higher-level ratio performance.

1.4. The Fibonacci asymmetry

There is an asymmetry in the Farey tree that is not evident in the Arnold tongues. Any ratio can be represented through the continued fraction method. The Fibonnaci sequence (identified by dashed lines in Fig. 3) corresponds to the one ratio at each Farey tree level that has the longest continued fraction representation. As such, Fibonacci ratios, more than other ratios at the same Farey tree level, most closely approximate the Golden Mean, $G = 0.61803$, which has the longest continued fraction representation of any number (Schroeder, 1991):

$$G = \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}} \tag{2}$$

G is converged upon as more fractions (to infinity) are added to the denominator of Eq. (2). In the generation of predictions about performance, the asymmetry imposed by the Fibonnaci sequence identifies a potential source of differences within each Farey tree level

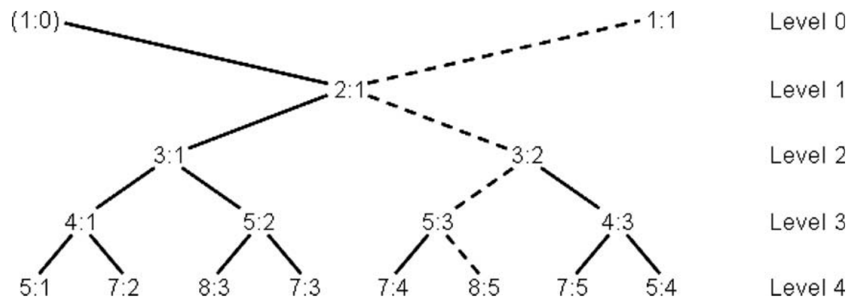

Fig. 3. Ratios from the first five levels (Levels 0–4) of the Farey tree are presented. Note that lower levels of the Farey hierarchy are depicted higher in the tree. Ratios along the dashed line are members of the Fibonacci sequence.

(Hessler & Amazeen, submitted for publication; Peper et al., 1991; Treffner & Turvey, 1993). However, the directionality of the Fibonacci asymmetry, specifically with respect to stability (variability) predictions, is controversial.

The leading hypothesis, known as the fraction substructure hypothesis, suggests that Fibonacci ratios should be less stable because of their longer continued fraction representations (Treffner & Turvey, 1993). According to Treffner and Turvey, performance of ratios with longer continued fraction representations is complicated because it requires more iterations. Whether Treffner and Turvey considered iterations to be mathematically abstract or manifested directly in biological processes, the result is slower convergence toward final frequency ratio values that theoretically renders performance more susceptible to disruption. In support of that interpretation, Treffner and Turvey observed that Fibonacci ratio performance was less stable than simple ratio (p:1) performance in bimanual coordination. However, the fraction substructure hypothesis is not consistent with the prevalence of Fibonacci ratios in nature. Among the more common observations are the ratio of volumes in consecutive chambers of the Nautilus shell (Cook, 1979) and the ratio of clockwise to counterclockwise spirals in the scales of pineapples (Livio, 2002) and the seeds of sunflower heads (Stewart, 1995).

The fact that continued fraction representations of Fibonacci ratios are generated through the iteration of a simple rule—add 1 to the lowest-order denominator term, in effect substituting the lowest-order 1 with $\frac{1}{1+1}$ —endows Fibonacci ratios with a fractal structure that non-Fibonacci ratios do not possess. It is that fractal structure that can be used to make the opposite argument that Fibonnaci ratios should be easier to perform than non-Fibonnaci ratios. The word fractal is based on the notion that fracturing a larger piece produces an infinite regress of smaller, identical pieces (Mandelbrot, 1977, 1982). That is, larger features nest within themselves smaller features that nest within themselves even smaller features (Van Orden, Holden, & Turvey, 2003). For example, in the Nautilus shell, the same spiral pattern is repeated, but smaller and smaller, over a range of scales (Cook, 1979). Although performance of Fibonacci ratios might require more iterations than other ratios at the same Farey tree level, performance of Fibonacci ratios is less complex because it involves repetition of one simple operation. One practical implication of such repetition (e.g., Van Orden et al., 2003) is that disturbances or even natural fluctuations at one level of the body (e.g., increased demand for oxygen) can be absorbed by fluctuations across larger (e.g., bodily movement) and smaller (e.g., circulation) levels.

1.5. Predictions

In the MRC literature, there is preliminary support that non-Fibonacci ratios are easier to perform than Fibonacci ratios that occupy Arnold tongues of the same width. At Level 3 of the Farey tree, 5:2, a non-Fibonacci ratio, can be performed naturally (e.g., Garlando et al., 1985), but 5:3, a Fibonacci ratio, has not been observed. We will further clarify interpretation of continued fraction structure when we ask participants to perform ratios from Levels 2 (*3:2* and 3:1), 3 (*5:3* and 5:2) and 4 (*8:5* and 8:3) of the Farey tree that are either members (italic typeface) or not members (normal typeface) of the Fibonacci sequence. The choice of ratios from different Farey tree levels provides a test of the hypothesis that performance of higher-level ratios will be less accurate and more variable than that of lower-level ratios.

Over two experiments, we compared performance on the above ratios using multiple display types: Lissajous, ball–ball, balloon–balloon, ball–balloon, perceptually manipulated and a static performance template. A no-display (control) condition provided a baseline comparison. We expected that the displays that added to the proprioceptive information already available to the performer (e.g., ball–balloon display) or that increased understanding of the required ratio (e.g., performance template) would enable greater performance accuracy and stability. Differences amongst veridical, perceptually manipulated and static performance template displays will clarify the nature of facilitation.

2. Experiment 1

In Experiment 1, motor-respiratory performance was compared across five display conditions: no display (control), a Lissajous display (e.g., Lee et al., 1995) and three ball and/or balloon feedback displays (Byblow et al., 1999; Wilson et al., 2005). Performance was expected to be better for the ball and/or balloon displays relative to the Lissajous display, because multifrequency Lissajous displays (e.g., Fig. 1B) may be too complex for participants to use effectively. The three ball and/or balloon displays (ball–ball, balloon–balloon, ball–balloon) varied with respect to perception-performance compatibility. We expected performance to be most accurate with the ball–balloon display because of greater compatibility between the display features and natural movement and breathing patterns.

2.1. Method

2.1.1. Participants

Fifty-five participants (27 men, 28 women; 18–50 years old; all right-handed by self-report) received either credit toward their introductory psychology course or $10 to participate. The participants had no arm, shoulder, or respiratory difficulties and did not smoke. Each participant was randomly assigned to a display condition. All participants were treated in accordance with the ethical principles of the American Psychological Association.

2.1.2. Apparatus

The participants sat in a chair and braces were placed on the elbow and wrist of their right arm to ensure rotation of the shoulder joint only. They swung their right arm forward and backward in the sagittal plane at a self-selected frequency and amplitude. A three-pound weight was held in order to make the self-selected frequency more consistent across participants. Infrared emitters were attached to a rigid piece of wood that was secured to the upper arm. An Optotrak/3020 (Northern Digital, Waterloo, Canada) that was positioned 2.2 m in front of the participants recorded arm movement. Breathing was recorded with a pneumotachometer (Hans Rudolph, Kansas City, MO), which samples air flow using a differential pressure method. The pneumotachometer was attached to a facemask that was worn over the nose and mouth. Arm movement and breathing data were sampled at 50 Hz and synchronized using an Optotrak Data Acquisition Unit.

2.1.3. Displays

The amplitude of arm movement and breathing data were analyzed in real time with an in-house Visual Basic program that produced feedback displays. The data were normalized to fit each display such that maximum excursion was similar across participants. The displays were projected on a wall 3 m in front of the participants with a high resolution computer projector.

Idealized *Lissajous* displays for 2:1 (Panel A) and 5:3 (Panel B) are presented in Fig. 1. Motion of a cursor (depicted as an open circle) produced a trace (refreshed every 10 s) that was specified by breathing (abscissa) and movement (ordinate). The participants were shown idealized Lissajous templates for each required ratio.

The *ball–ball* (Panel A), *balloon–balloon* (Panel B) and *ball–balloon* (Panel C) displays are presented in Fig. 4. Upward–downward motion of balls and expansion–contraction of balloons were controlled by either breathing (presented in blue to participants, labeled B in Fig. 4) or arm movement (presented in red to participants, labeled M in Fig. 4). The horizontal distance between balls and/or balloons was 40 cm from center to center. Each ball was 15 cm in diameter. Ball excursion occurred along the paths depicted by the vertical dashed lines (range ≈ 100 cm). Balloon excursion ranged from a pixel when fully deflated to approximately 25 cm (dotted lines).

2.1.4. Procedure

To familiarize themselves with ratio performance and the display condition, the participants practiced a 2:1 ratio between movement and breathing for 60 s. They then performed two 60 s trials for each of the six ratios from Levels 2 (*3:2* and 3:1), 3 (*5:3* and 5:2) and 4 (*8:5* and 8:3) of the Farey tree (randomized presentation), half of which were (italic typeface) and half of which were not (normal typeface) members of the Fibonacci sequence. There was a minimum 30-s rest between each trial and, upon request, more rest was provided.

2.1.5. Dependent measures

We used two methods to estimate the frequency ratio: (1) point (calculated cycle-by-cycle); and (2) power spectrum (calculated over the whole trial). An advantage of the point estimate is that it allows for the calculation of variability over all the observed frequency ratios in each trial. A disadvantage is that the point estimate may be vulnerable to temporal measurement error, particularly for more variable signals. An advantage of the power spectrum estimate is that it is less sensitive to temporal measurement error because entire signals are accounted for in its calculation. A disadvantage is that frequency ratio variability cannot be calculated using the power spectrum estimate because it is a composite measure derived from separate power spectra for movement and breathing.

Movement and breathing frequencies were first calculated to determine the *point estimate* of the frequency ratio (Amazeen et al., 2001; Peper et al., 1991, 1995a, 1995b) by dividing the 50 Hz sampling rate by the difference between successive movement maxima (forward-most arm movement positions) and successive inhalation maxima, respectively. We used movement and inhalation maxima, which were well-defined, to reduce the vulnerability of the point estimate to temporal measurement error. The point estimate was calculated by dividing movement frequency by breathing frequency at the location of each inhalation maximum. This resulted in a cycle-by-cycle frequency ratio estimate for each trial.

Movement and breathing power spectra were produced to determine the *power spectrum estimate* of the frequency ratio: the fast Fourier transform was used to decompose movement and breathing data into their component frequencies. Power, the squared magnitude at each frequency, was then determined. For some power spectra, there was more than one prominent peak. We calculated a weighted average of frequencies for each power spectrum. This calculation does not affect the frequency estimates for power spectra in which there is only a dominant peak but allows for the contribution of other prominent peaks when they exist. The power spectrum estimate was the ratio of the weighted movement frequency and breathing frequency.

Frequency ratio accuracy (absolute error (AE)) was calculated for the point (AE_{point}) and power spectrum (AE_{power}) estimates by taking the average absolute difference between the observed frequency ratio and the intended frequency ratio. Frequency ratio var-

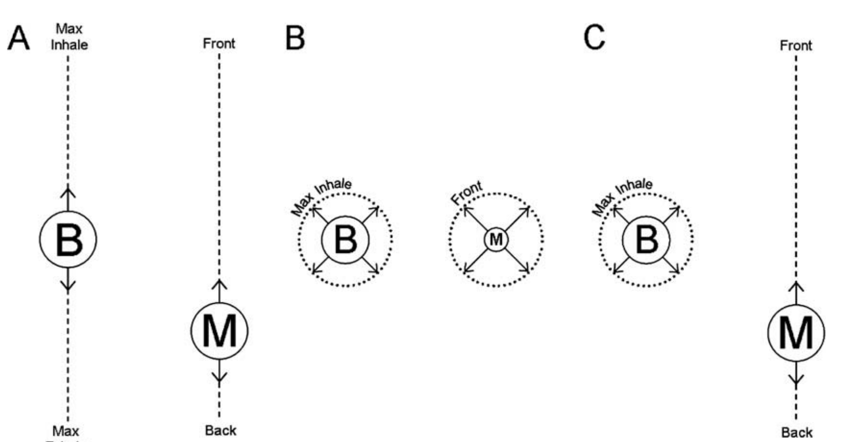

Fig. 4. Veridical ball and/or balloon displays include the ball–ball display (A), the balloon–balloon display (B) and the ball–balloon display (C). Balls/balloons were controlled either by breathing (labeled B) or movement (labeled M). Vertical dashed lines represent the total excursion of the balls. Dotted circles represent the maximum expansion of the balloons.

iability (variable error (VE)) was calculated by taking the standard deviation of all point estimates of the frequency ratio within a trial (VE_{point}).

2.1.6. Design

Duplicate trials were collected to ensure one analyzable trial per ratio. Dependent measures from the first trial on each ratio were analyzed with 5 (Display: Lissajous, control, balloon–balloon, ball–ball, and ball–balloon) $\times$ 3 (Farey Tree Level: two, three, and four) $\times$ 2 (Fibonacci Condition: non-Fibonacci and Fibonacci) mixed analyses of variance (ANOVAs) with Display as a between-subjects factor and Farey Tree Level and Fibonacci Condition as within-subjects factors.

2.2. Results

2.2.1. Frequency ratio performance

Fig. 5 depicts the observed frequency ratios as a function of Display Type, Farey Tree Level, and Fibonacci Condition. Each symbol is the point estimate of the frequency ratio for a trial. The pattern of results was similar for the power spectrum estimate of the frequency ratios. There was a distribution of observed ratios around each intended ratio. Accuracy (AE) can be gauged by comparing the observed ratio to each intended ratio. Table 1 depicts AE_{point} and AE_{power} across all display types for the non-Fibonacci and Fibonacci ratios. The effect of Display was significant on both estimates of AE (AE_{point}: $F(4, 50) = 3.41$, $p < .05$; AE_{power}: $F(4, 50) = 3.93$, $p < .01$). Ball and/or balloon display performance was generally quite accurate. The effect of Fibonacci Condition was significant on both estimates of AE (AE_{point}: $F(1, 50) = 4.24$, $p < .05$; AE_{power}: $F(1, 50) = 9.83$, $p < .01$). Performance accuracy was greater for Fibonacci than for non-Fibonacci ratios. No other main effects or interactions were significant for AE_{point} or AE_{power}.

Contrasts were performed between all display pairs. Performance with the ball and/or balloon displays was more accurate than performance with the Lissajous display (AE_{point}: balloon–balloon vs. Lissajous, $F(1, 50) = 7.27$, $p < .01$, ball–ball vs. Lissajous, $F(1, 50) = 7.57$, $p < .01$, ball–balloon vs. Lissajous, $F(1, 50) = 9.44$, $p < .01$; AE_{power}: balloon–balloon vs. Lissajous, $F(1, 50) = 10.02$, $p < .01$, ball–ball vs. Lissajous, $F(1, 50) = 6.98$, $p < .05$, ball–balloon vs. Lissajous, $F(1, 50) = 10.81$, $p < .01$). Of performance with the ball and/or balloon displays, only performance with the ball–balloon display was marginally more accurate than performance in the control condition (AE_{point}: $F(1, 50) = 3.56$, $p = .066$; AE_{power}: $F(1, 50) = 3.88$, $p = .054$). Performance with the Lissajous display did not significantly differ from performance in the control condition.

Table 1
Mean frequency ratio absolute error as a function of Display Type and Fibonacci Condition.

Display Type	Point	Power spectrum
Lissajous		
Non-Fibonacci	0.75	0.83
Fibonacci	0.61	0.64
Control		
Non-Fibonacci	0.59	0.62
Fibonacci	0.48	0.54
Balloon/balloon		
Non-Fibonacci	0.40	0.47
Fibonacci	0.28	0.24
Ball/ball		
Non-Fibonacci	0.38	0.56
Fibonacci	0.29	0.29
Ball/balloon		
Non-Fibonacci	0.31	0.37
Fibonacci	0.28	0.32

Fig. 6 depicts VE_{point} for non-Fibonacci and Fibonacci ratios from Levels 2–4 of the Farey tree. The effect of Farey Tree Level was significant on VE_{point}, $F(2, 100) = 6.28$, $p < .01$. Within-subjects contrasts were performed between all Farey tree levels. Performance variability was significantly lower for ratios at Level 2 than at Level 3, $F(1, 50) = 8.69$, $p < .01$, or Level 4, $F(1, 50) = 7.24$, $p < .01$, but did not differ at Levels 3 and 4 ($p = 0.83$). The effect of Fibonacci Condition was significant on VE_{point}, $F(1, 50) = 22.19$, $p < .001$. Performance variability was lower for Fibonacci than non-Fibonacci ratios. Combined with the AE results, performance of Fibonacci ratios was more accurate and less variable than performance of non-Fibonacci ratios. No other main effects or interactions were significant for VE_{point}.

2.2.2. Movement frequency

Movement frequencies were not different amongst the performance of Level 2 ($M = 0.675$ Hz), Level 3 ($M = 0.687$ Hz) and Level 4 ($M = 0.690$ Hz) ratios, $F(2, 100) < 1$, nor between the performance of Fibonacci ($M = 0.684$ Hz) and non-Fibonacci ($M = 0.684$ Hz) ratios, $F(1, 50) < 1$. Therefore, faster movement frequencies do not explain the increases in variability at higher Farey tree levels and for non-Fibonacci ratios.

2.3. Discussion

In this experiment, we examined the effect of different displays on MRC performance. Performance accuracy was significantly lower with the Lissajous display relative to the ball and/or balloon dis-

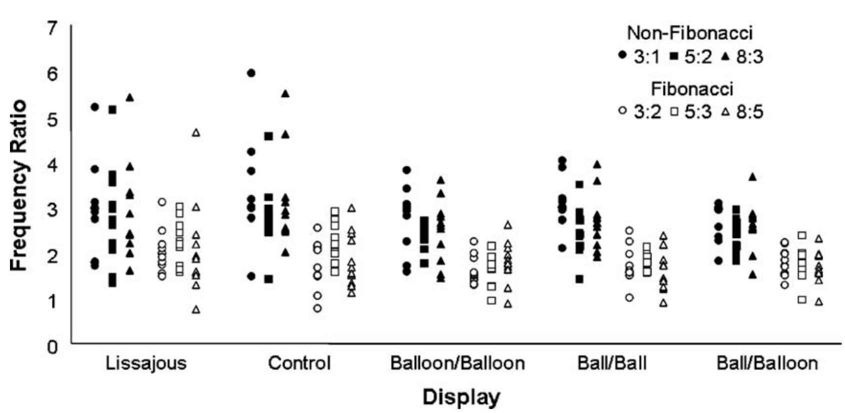

Fig. 5. The observed frequency ratios as a function of Display Type, Farey Tree Level, and Fibonacci Condition. Each symbol is the point estimate of the frequency ratio for a trial. The decimal equivalents for the intended ratios were 3:1 = 3.00, 3:2 = 1.50, 5:2 = 2.50, 5:3 = 1.67, 8:3 = 2.67, and 8:5 = 1.60.

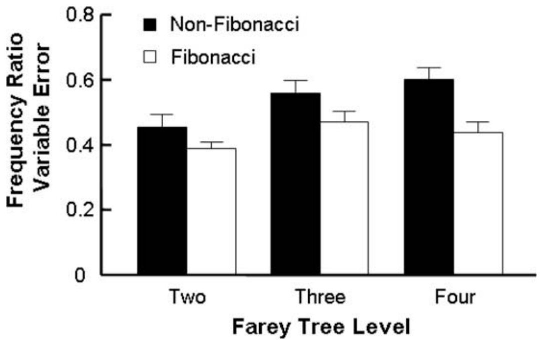

Fig. 6. Mean frequency ratio variable error (VE_{point}) as a function of Farey Tree Level and Fibonacci Condition.

plays. This result differs from previously observed results in which Lissajous displays benefited 1:1 (Hurley & Lee, 2006; Lee et al., 1995; Wenderoth & Bock, 2001) and 2:1 (Swinnen et al., 1997) bimanual coordination. There was no Display × Farey Tree Level interaction on performance accuracy, which suggests that the Lissajous displays did not even help performance of commonly observed ratios (e.g., 3:1). If the Lissajous display is used for MRC, then that result implies that it would not assist in the performance of ratios more complex than 2:1. Lissajous displays may be helpful only to the extent that they simplify task information.

Veridical ball and/or balloon displays included upward–downward and/or expansion–contraction motion to assess perception-performance compatibility (Byblow et al., 1999; Shepard & Hurwitz, 1984; Wilson et al., 2005). The ball–balloon display was developed to be most compatible with the component activities of MRC and included upward–downward motion for movement and expansion–contraction motion for breathing. Across a range of ratios, only performance with the ball–balloon display was better than performance with no display. This finding supports the notion that perception-performance compatibility is a desirable feature of feedback displays.

MRC performance, as reflected in the level results, was consistent with sine circle map predictions and previous bimanual coordination results (de Guzman & Kelso, 1991; Deutsch, 1983; Haken et al., 1996; Peper et al., 1991, 1995a, 1995b; Treffner & Turvey, 1993). Performance of ratios from Level 2 was more stable than from Levels 3 and 4. Support for the beneficial nature of continued fraction structure was found in the greater performance accuracy and stability of Fibonacci ratios in comparison to non-Fibonacci ratios. This result replicates and extends findings from bimanual coordination to MRC (de Guzman & Kelso, 1991; Kelso & de Guzman, 1988). Together, the results of Experiment 1 suggest that the facilitating nature of feedback displays takes place against a background of natural dynamics specified for multifrequency coordination by the sine circle map and continued fraction structure.

3. Experiment 2

The display that best facilitated performance in Experiment 1, as evidenced by greater performance accuracy relative to a no-display (control) condition, was the ball–balloon display (see Fig. 4C). This display was compared to the perceptually manipulated (Fig. 1C) and performance template (Fig. 1D) displays in Experiment 2, which were designed to alleviate the perceptual or conceptual requirements of MRC, respectively. Both the ball–balloon and perceptually manipulated displays simplified the perceptual task because they provided real-time, augmented feedback. However, performers faced a unique conceptual challenge with each display: the ball–balloon display required a counting strategy and the perceptually manipulated display required an understanding of how proprioceptive feedback related to changes in the display. In contrast, the performance template was perceptually challenging, due to the lack of augmented feedback, but was conceptually simplified.

A pattern of results in which performance is most accurate with the two feedback displays will imply that difficulty in MRC performance lies in the self-perception of motor and respiratory activity. Any observed differences between performance with the two feedback displays will identify whether perception-performance compatibility or perceptual simplification is the more desirable feature for feedback. If performance is most accurate with the performance template, then the implication is that the participants must have adequate proprioceptive feedback but need the assistance of a template to understand the pattern that is to be performed.

3.1. Method

3.1.1. Participants

Sixty participants (28 men and 32 women; 18–45 years old; all right-handed by self-report) received credit toward their introductory psychology course or $10 to participate. Each participant was randomly assigned to a display condition.

3.1.2. Apparatus and displays

The apparatus was identical to that used in Experiment 1. Displays were produced and projected as described in Experiment 1, with one exception: the performance template was produced using PowerPoint.

The *ball–balloon* display was the same as that used in Experiment 1 (Fig. 4C).

The *perceptually manipulated* display is depicted in Fig. 1C. Rotation of an 80 cm line from horizontal specified the magnitude and direction of error of the ratio between movement and breathing (performed ratio − intended ratio). Ratio error was calculated at the completion of each breathing cycle. The display was, therefore, refreshed at the completion of each breathing cycle. As a reference, if the difference between the performed ratio and the intended ratio equaled 0.5, then the line rotated 18° above horizontal. Maximum rotation was ±60°.

A *performance template* for 3:2 is depicted in Fig. 1D; separate templates were produced for every ratio. Performance templates were composed of horizontal lines for arm movement and breathing with hash marks that identified forward-most arm movements or inhalation onsets.

3.1.3. Procedure

The procedure was identical to that used in Experiment 1 except that a subset of ratios was performed from Levels 2 (*3:2* and 3:1) and 4 (*8:5* and 8:3) of the Farey tree that were (italic typeface) and were not (normal typeface) members of the Fibonacci sequence. In Experiment 1, variability in the performance of Levels 3 and 4 ratios was comparable. Therefore, Level 3 ratios were eliminated from the present experiment to increase statistical power.

3.1.4. Dependent measures and design

Dependent measures were identical to those used in Experiment 1. Each was analyzed with 4 (Display: control, ball–balloon, perceptually manipulated, and performance template) × 2 (Farey Tree Level: two and four) × 2 (Fibonacci Condition: non-Fibonacci and Fibonacci) ANOVAs with Display as a between-subjects factor and Farey Tree Level and Fibonacci Condition as within-subjects factors.

3.2. Results

3.2.1. Frequency ratio performance

Fig. 7 depicts the observed frequency ratios as a function of Display Type, Farey Tree Level, and Fibonacci Condition. As in Fig. 5 (Experiment 1), the point estimate is depicted but the pattern of results was similar for the power spectrum estimate. The accuracy (AE) of performance can be gauged by comparing observed ratios to each intended ratio. Table 2 depicts AE_{point} and AE_{power} for all display types. The effect of display was significant on both estimates of AE (AE_{point}: $F(3, 56) = 4.16$, $p < .05$; AE_{power}: $F(3, 56) = 5.03$, $p < .01$). Displays ordered such that performance accuracy was greatest with the performance template, roughly equivalent for the two real-time feedback displays, and worst with no display. No other main effects or interactions were significant for AE_{point} or AE_{power}.

Contrasts were performed between all display condition pairs. Performance accuracy was significantly greater with the performance template in comparison with the control condition (AE_{point}: $F(1, 56) = 12.38$, $p < .001$; AE_{power}: $F(1, 56) = 14.74$, $p < .001$) and the real-time feedback displays (AE_{point}: performance template vs. ball–balloon, $F(1, 56) = 4.00$, $p = .051$, performance template vs. perceptually manipulated, $F(1, 56) = 4.35$, $p < .05$; AE_{power}: performance template vs. ball–balloon, $F(1, 56) = 5.83$, $p < .05$, performance template vs. perceptually manipulated, $F(1, 56) = 5.03$, $p < .05$).

Fig. 8 depicts VE_{point} for non-Fibonacci and Fibonacci ratios from Levels 2 and 4 of the Farey tree. The effects of Farey Tree Level, $F(1, 56) = 4.52$, $p < .05$, and Fibonacci Condition, $F(1, 56) = 5.74$, $p < .05$, were significant on VE_{point}. Performance variability was lower for Level 2 than Level 4 ratios and lower for Fibonacci than non-Fibonacci ratios. Both findings replicate the results of Experiment 1. No other main effects or interactions were significant for VE_{point}.

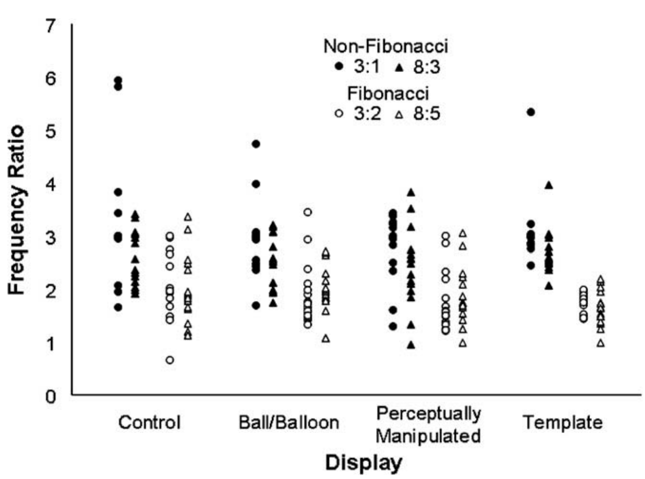

Fig. 7. The observed frequency ratios as a function of Display Type, Farey Tree Level, and Fibonacci Condition. Each symbol is the point estimate of the frequency ratio for a trial. The decimal equivalents for the intended ratios were 3:1 = 3.00, 3:2 = 1.50, 8:3 = 2.67, and 8:5 = 1.60.

Table 2
Mean frequency ratio absolute error as a function of Display Type.

Display Type	Point	Power spectrum
Control	0.62	0.63
Ball/balloon	0.46	0.49
Perceptually manipulated	0.47	0.48
Performance template	0.25	0.26

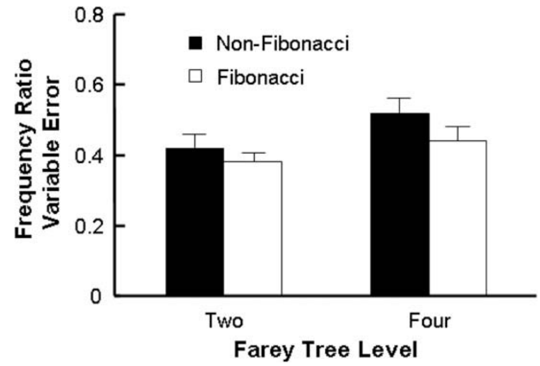

Fig. 8. Mean frequency ratio variable error (VE_{point}) as a function of Farey Tree Level and Fibonacci Condition.

3.2.2. Movement frequency

As in Experiment 1, we analyzed movement frequency to evaluate if more variable performance could have resulted from higher movement frequencies. The main effect of Farey Tree Level on movement frequency, $F(1, 56) = 5.14$, $p < .05$, was significant. Level 4 ratios were performed at a higher movement frequency (M = 0.707 Hz) than Level 2 ratios (M = 0.689 Hz). This increase in movement frequency is a possible explanation for the observed effect of Farey Tree Level on variability. However, this variability effect was more likely a result of the Level manipulation given that the increase in movement frequency was a rather modest 0.018 Hz and movement frequencies did not differ across levels in Experiment 1. To be more certain in future studies, movement frequency could be paced using a metronome. Movement frequencies were not different between Fibonacci (M = 0.696 Hz) and non-Fibonacci (M = 0.700 Hz) ratio performance, $F(1, 56) < 1$. The observed effect of Fibonacci Condition on variability cannot be explained by faster movement frequencies.

3.3. Discussion

This experiment was designed to compare displays that enhanced either perceptual feedback or ratio conceptualization. Performance was not enhanced with the perceptual feedback displays relative to the no-display (control) condition, which indicates that perceptual feedback provided no additional benefit to multifrequency performance. Ratio performance was more accurate with the simple, static performance template in comparison to the dynamic feedback displays. This result is consistent with the benefits of the performance template for learning in MRC (Hessler & Amazeen, submitted for publication) and training in bimanual coordination (Summers et al., 1993). The benefit of the performance template is likely to arise from presentation of the relative timing of movement and breathing landmarks. During spontaneous MRC, the challenge may be that the relative timing information needed to perform more difficult ratios is not available. The participants benefited when the visual depiction of timing needed to support intentional ratio performance was available.

The results of Experiment 2 replicated the findings of Experiment 1 that MRC performance was consistent with the level predictions depicted in the Farey tree and our alternate interpretation of the continued fraction structure. Performance variability was reduced at lower Farey tree levels and was smaller for Fibonacci than for non-Fibonacci ratios. Displays facilitated performance of ratios within a system of natural constraints as specified by the sine circle map. To the extent that sine circle map principles generalize across both MRC and bimanual coordination suggests that the multifrequency dynamics observed are independent of the particular physiological subsystem that produces the coordination pattern.

4. General discussion

Only a small subset of ratios has been observed during various exercises (e.g., Amazeen et al., 2001; Bramble & Carrier, 1983; Garlando et al., 1985; Mahler, Hunter et al., 1991; van Alphen & Duffin, 1994), but we suspected that performers might be able to produce a greater number of ratios with assistance. Over the course of two experiments and using six different visual displays, the individuals were able to perform a number of ratios without prior practice that had not been observed previously. The features of those displays that facilitated performance were informative with respect to the challenges involved in multifrequency ratio production and will be discussed below in the context of the control of action.

4.1. Display features

In Experiment 1, performance accuracy was greater with the ball and/or balloon feedback displays relative to the Lissajous display, but only with the ball–balloon display was performance accuracy greater than with the no-display (control) condition. Like the Lissajous display, the ball–balloon display provided augmented feedback but in a marginally more effective format. The motion in the ball–balloon display was designed to map more compatibly with the performance characteristics of movement and breathing. The participants seemed not to be negatively affected by the requirement that they count the number of cycles for each component of the ball–balloon display. However, there may be an upper limit to the complexity of the ratio (e.g., 17:5) that can be performed when counting is required.

Perceptual manipulation of visual feedback in Experiment 2 neither further benefited performance nor hurt performance relative to the ball–balloon display. The fact that the participants do not need to know the details of the required pattern may make the perceptually manipulated display methodologically useful. Two possibilities include frequency-induced phase transition (Haken, Kelso, & Bunz, 1985; Peper et al., 1995b) and perturbation (e.g., Kelso, Schöner, Scholz, & Haken, 1987) experiments, in which ratios may be changed, either permanently or temporarily and without the performer's explicit knowledge, in order to probe ratio stability. Thus, testing a large range of ratios with the perceptually manipulated display may enable the empirical ordering of ratios according to their performance stability.

The performance template produced the most accurate performance of all ratios in this study. In contrast to the other displays that provided perceptual feedback, the performance template provided information about required relations between movement and breathing landmarks. Naturally available proprioceptive information was apparently sufficient for performers to control the activities of the motor and respiratory subsystems of the body, but the MRC task itself, which required control over relations between those physiological subsystems, produced a greater need for conceptual information than for perceptual assistance. The presentation of additional information in the performance template likely allowed the participants to situate starting ratios within the appropriate Arnold tongues. As specified in the sine circle map, once that is achieved, performance would easily evolve toward the instructed rational ratio.

The finding that the participants could perform novel ratios with the performance template suggests that the potential exists for a larger behavioral repertoire. However, current evidence suggests that the reason for there being a smaller repertoire is that the lower-level ratios athletes perform naturally during aerobic exercise are very efficient (Bernasconi & Kohl, 1993; Bonsignore, Morici, Abate, Romano, & Bonsignore, 1998; Daffertshofer, Huys, & Beek, 2004). For example, Daffertshofer et al. proposed a dynamical model of MRC based on observations from rowing. During the rowing stroke, the lungs are periodically compressed (Siegmund et al., 1999). Therefore, it is desirable to inhale between compressions. This is achieved most effectively and oxygen consumption is maximized when movement and breathing lock into specific lower-level frequency ratios. As such, naturally available ratios are likely to be more efficient, at least for aerobically demanding exercises, than the novel ratios performed in this study.

Flexibility allows athletes to accommodate to changing performance demands. Previous studies have documented natural ratio shifts with increases in movement frequency during running (Bramble & Carrier, 1983) and wheelchair propulsion (Amazeen et al., 2001), as well as intentional control over the ratios used in order to control gear shifts during cycling (Garlando et al., 1985). A means of increased flexibility, proposed by Garlando et al., is that the coupling strength between movement and breathing should not be too high for any one ratio so that athletes can easily switch between those ratios that are naturally available. In the current study, the use of performance templates benefited performance of novel ratios. In future studies, such templates could be used to train athletes to become more proficient at switching between intrinsically available ratios.

4.2. Natural constraints are captured by dynamical principles

The finding that visual displays could be used to expand the range of performed ratios has implications for model testing. Amazeen et al. (2001) hinted at applications of the sine circle map to MRC, but any predictions about complex ratio performance could not be tested until a method was developed by which to elicit performance of multiple complex ratios within the same experimental session from the same individual. Development of displays in the present study made possible the experimental design necessary for a test of sine circle map predictions. We tested two main predictions in the present study, although the previous discussion of the perceptually manipulated display hints at other experimental designs that are now possible as well. Those two predictions were that (1) performance of lower-level ratios is more stable than performance of higher-level ratios and (2) an asymmetry exists between ratios that are and are not members of the Fibonacci sequence that has implications for performance stability.

The finding that performance of lower-level ratios was more stable than performance of higher-level ratios supports previous research in both MRC (Amazeen et al., 2001; Villard et al., 2005) and bimanual coordination (de Guzman & Kelso, 1991; Deutsch, 1983; Haken et al., 1996; Peper et al., 1991, 1995a, 1995b; Treffner & Turvey, 1993). Although the observed patterns of stability across levels were interpreted in terms of the sine circle map, those observed patterns are also largely consistent with the predictions that can be reached with the Summers et al. (1993) timekeeper model. That is, there were more opportunities for the participants to misidentify breathing locations and timing was more complex (e.g., breaths were required during multiple movement beats and with varying time delays) for the higher-level ratios that were performed in this study.

The finding that performance of Fibonacci ratios was more stable than performance of non-Fibonacci ratios was inconsistent with the bimanual coordination results of Treffner and Turvey (1993) but supports a cost-efficient interpretation of continued fraction structure. The Fibonacci sequence identifies one ratio per Farey tree level that has the longest continued fraction representation. Treffner and Turvey interpreted longer continued fraction representations as evidence for less stable performance of Fibonacci ratios. A comparison of Fibonacci ratios with simple ratios at the same Farey tree level supported their hypothesis. The fact that simple ratios are overwhelmingly preferred in MRC motivated us to compare complex Fibonacci ratios to non-Fibonacci ratios, which (above Level 2) were also complex. The implication of our results is that longer continued fraction representations, which are generated through the iteration of one simple operation, actually benefit ratio performance.

5. Conclusion

At its simplest level of interpretation, the present study was designed to facilitate performance of MRC patterns that are rarely, if ever, observed during natural activities. However, at a deeper level of interpretation, the results offer theoretical insight into the perceptual and conceptual challenges for the control of action. The performers were sufficiently attuned to their body's natural tendencies to control their actions, even to the extent that control required the synchronization of activities across multiple subsystems of the body and in patterns that have not been observed naturally. Where performers required assistance was in the conceptualization of the required pattern, particularly with respect to information about the relative timing of relevant landmarks. Whether continuous access to that information is necessary to maximize control remains an empirical question, but it will clarify the timing mechanisms for this physiologically important and theoretically revealing between-systems coordination task.

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